Continuity: The stream-tube area perpendicular to the direction of flow may be expressed by

$$A_{st} = A_{st,\,perp}[V_a^2/(V_t^2 + V_a^2)]^{1/2}$$

where $A_{st,\,perp}$ is defined as before. Therefore, the continuity equation may be written two ways (neglecting V_r) as

$$\dot{m}_{st} = \rho A_{st}(V_a^2 + V_t^2)^{1/2} = \rho\, A_{st,\,perp}\, V_a \tag{7}$$

Euler equation along a stream tube: This may be written, neglecting initial velocities in comparison to velocities in the throat region (as done in simplified one-dimensional treatments), and neglecting radial velocity, as

$$\int_{P_0}^{P} \frac{dp}{\rho} + \frac{V_t^2 + V_a^2}{2} = 0 \tag{8}$$

Isentropy:

$$P/\rho^\gamma = \text{const} = K \tag{9}$$

Equations (4, 5, and 7–9) may be combined to yield an expression for $[\dot{m}_{st}/A_{st,\,perp}]$. Since $A_{st,\,perp}$ is minimized at the throat, $[\dot{m}_{st}/A_{st,\,perp}]$ must be maximized there. Application of this criterion (the "choking criterion") by differentiation of the expression for $\dot{m}_{st}\, A_{st,\,perp}$ with respect to pressure and setting the resulting expression equal to zero yields the throat pressure for each streamline,

$$P_* = \left[\left(\frac{2}{\gamma + 1}\right) P_0^{(\gamma-1)/\gamma} - \frac{(\gamma - 1)K^{-1/\gamma}}{(\gamma + 1)\,\gamma}\, \omega_0^2 \left(\frac{R_0}{R^*}\right)^4 r^2\right]^{\gamma/(\gamma-1)} \tag{10}$$

which upon suitable algebraic manipulation yields Eq. (3) for the square of the axial velocity. Still more mathematical exercise shows that the axial Mach number (V_a/a) at the physical throat is 1.0, independent of the radial position while the resultant Mach number $[(V_a^2 + V_t^2)^{1/2}/a]$ passes through 1.0 before the physical throat, everywhere except at the centerline. This results from the fact that, as discussed earlier, flow in a stream tube is not perpendicular to the "hardware" cross-sectional area; and thus the fluid in a given stream tube may see a minimum area perpendicular to the flow direction upstream of the physical throat. The mathematical detail presented here is quite skimpy because of length restrictions, but can be greatly amplified on request.

But now let us examine Eq. (10) a bit more carefully. A small amount of algebraic manipulation or numerical substitution shows that the radial Navier-Stokes equation cannot be satisfied simultaneously in the motor chamber and at the throat. Let us recall that the radial Navier-Stokes equation was not employed in the analysis.

Let us next turn to Manda's analysis.[1] He assumes conservation of angular momentum and proportional tapering of his gas "rings." These assumptions may be shown to be entirely equivalent to the assumptions of conservations of angular momentum and maintenance of an ideal forced vortex pattern. He then uses a form of the equation of conservation of energy that is equivalent to the use of the Euler equation, the isentropy relationship, and the ideal gas law, along each stream tube in combination with the radial Navier-Stokes equation at the throat (using the one-dimensional, nonswirl case to obtain a boundary condition at the centerline since the tangential velocity is zero at the centerline) in order to solve for the axial velocity at the throat, V_{a_*}. In the application of the energy equation, he does not neglect the initial tangential velocity as this author does in application of the Euler equation. This, however, should lead to only a small difference in results and is not important to the discussion. It should be noted that nowhere has the concept of choked flow been brought in and that, in fact, according to the analysis of this author, this criterion has not been satisfied.

Finally, let us look at Bastress' treatment.[2] He has likewise made the assumptions of conservation of angular momentum and maintenance of a "spinning-plug" flow (or, equivalently, uniform contraction of the stream-lines). However, he has additionally assumed the radial pressure gradient to be everywhere zero. Finally, he essentially assumed that the temperature, pressure, and density will be the same at the throat as in the nonspin case when he assumed that the sum of the squares of the axial and tangential velocities in the spin case is equal to the square of the axial velocity in the nonspin case. The final result satisfies neither the radial Navier-Stokes equation nor the "choke criterion."

Conclusions

By now, it should be seen that the solutions have one problem in common; that is, they do not satisfy simultaneously all of the governing equations which must be applied to them. Bastress' system is seriously overspecified; those of Manda and this author are overspecified by only one degree. In the one-dimensional nonswirl case, there are basically five unknowns (T, P, ρ, V_a, $\dot{m}$) and four equations (Euler, continuity, the ideal gas law, and equivalently either the energy equation or the isentropic relationship) plus the choke criterion. When swirling flow is brought in, the additional variables of ω and V_t enter. However, if conservation of angular momentum and uniform tapering of stream sheets (or equivalently, maintenance of a forced vortex pattern) are required, these criteria, along with the radial Navier-Stokes equation, provide three new independent equations. Thus, the problem is overspecified. Therefore, it appears to this author that the assumption of maintenance of a forced vortex flow pattern must be dropped for accurate solution of the problem of flow of a fluid from a chamber in which it is swirling through a converging nozzle.

References

[1] Manda, L., "Spin effects on rocket nozzle performance," J. Spacecraft Rockets **3**, 1695–1696 (1966).

[2] Bastress, E. K., "Interior ballistics of spinning solid-propellant rockets," J. Spacecraft Rockets **2**, 455–457 (1965).

Reply by Author to M. King

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Both King's analysis and the author's are attempts to approximate the solution of the inviscid Navier-Stokes equations for axisymmetric subsonic flow with rotation. In both efforts, velocities in the radial direction (V_r) are assumed negligible. Therefore, with axial symmetry ($\partial/\partial\theta = 0$) and $V_r = 0$, the equations of motion in polar coordinates can be reduced to

$$(1/\rho)(\partial P/\partial x) = -V_a(\partial V_a/\partial x) \tag{1}$$

$$(1/\rho)(\partial P/\partial \theta) = 0 = -V_a(\partial/\partial x)(rV_t) \tag{2}$$

$$(1/\rho)(\partial P/\partial r) = (V_t^2/r) \tag{3}$$

which are applicable throughout the flowfield.

In addition, the conservation of mass (in any annular stream-tube) requires that

$$\rho V_a(\partial A/\partial x) + A(\partial/\partial x)(\rho V_a) = 0 \tag{4}$$

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where $\partial A/\partial x = 0$ at the nozzle throat, thus yielding King's choking criterion if both $\partial \rho/\partial x$ and $\partial V_a/\partial x$ are finite at the throat section.

However, the variation of both ρ and V_a in the axial (x) direction at the nozzle throat can be reduced to zero with a well-rounded approach section, and this is the assumption made in the author's analysis, thus satisfying criterion (4).

With $\partial P/\partial x = \partial P/\partial \theta = 0$ at the nozzle throat, Eq. (3) is transformed to a total differential equation, with the solution given by the author's Eq. (10) if it is assumed that $r_*/r_0 = R_*/R_0$. The validity of this assumption is open to question, but hopefully it should suffice for a first approximation.

It can be shown that King's Eq. (10) will result in a radial pressure gradient given by

$$(1/\rho)(\partial P/\partial r) = -[2/(\gamma + 1)](V_t^2/r)$$

which fails to fulfill the requirements of Eq. (3) in both magnitude and direction.

Thus, the author's results are found to be compatible with each of the governing criteria, Eqs. (1–4), whereas King's results fail to satisfy the radial momentum balance required by Eq. (3) and must therefore be considered invalid.

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Comment on Probe Payload Selection

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DETERMINATION of the scientific payload of a Mars probe (or any other probe) involves choosing one of the many possible combinations of instruments that do not exceed specific weight, power, and required bandwidth constraints. A decision problem of this nature is typically formulated in the following manner[1]: 1) List the possible actions, $A_1 \ldots A_n$ (in this case the possible combinations of instruments); 2) List the alternative possible environments to be encountered, $E_1 \ldots E_m$; and 3) Assign a utility U_{ij} to the outcome of having chosen act A_i if environment E_j occurs for all combinations of A and E. (The U_{ij}s are "cardinal" utilities, unique up to a linear transformation, and usually normalized to the interval between zero and one.) Decision theory attempts to provide rules for choosing an optimum action in the face of uncertainty. The basic point is that the expected utility of each set of instruments can depend critically on the environment encountered. Hence, the relative likelihoods of the alternative environments can strongly affect the choice of instruments. It is somewhat surprising, therefore, to find that in recent studies of Voyager payload selection by Fosdick and Morgenthaler,[2] Dyer,[3] and Puette,[4] *there is no mention of the effect that uncertainty in the enviornment would have on instrument selection.*

The conclusion these authors draws from the foregoing is threefold: 1) Previous "objective" payload optimizations are at best misleading since they fail to consider uncertainty in the environment to be encountered. 2) Quantitative payload optimization is currently impossible since there exist no techniques for maximizing expected utility when the probability distribution over possible states of nature is not precisely known.[5] But, 3) even if quantitative payload optimization is impossible, explicit consideration of alternative environments, as well as alternative payloads, can sharpen our thinking about decisions that will remain for some time a matter for qualitative judgement.

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References

[1] Luce, R. and Raiffa, H., *Games and Decisions: Introduction and Critical Survey* (John Wiley & Sons Inc., New York, 1957) Chaps. 2, 13, pp. 12–38 and 275–326.

[2] Fosdick, G. and Morgenthaler, G., "Unique mars capture probe and probe payload selection," J. Spacecraft Rockets **3**, 142–145 (1965).

[3] Dyer, J., "Selection of scientific payloads for deep space exploration," *Joint Meeting of the Institute of Management Sciences and the Operations Research Society of America* (Minneapolis, Minn., 1964) pp. 1–17.

[4] Puette, R., "Obtaining a priority listing of scientific experiments for Voyager," *Stanford Advanced Mars Project for Life Detection, Exploration, and Research* (SAMPLER), edited by J. Kiely, Puette, R., Jamison, D., and Baker, G. (Stanford University, Stanford Calif., 1965), pp. 478–489.

[5] Luce, R. and Suppes, P., "Preference, utility, and subjective probability," *Handbook of Mathematical Psychology*, Vol. III, edited by R. Luce, R. Bush, and E. Galanter (John Wiley & Sons, Inc., 1965), New York, Chap. 19, pp. 299–306.

Erratum: "Effects of Atmospheric Drag on the Position of Satellites in Eccentric Orbits"

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[J. Spacecraft and Rockets **3**, 547–553(1966)]

IN this paper, Eq. (49) should read

$$P(2\pi n + \alpha, \epsilon) = 2\pi^2 n^2 I_0(\epsilon) + P(\alpha, \epsilon) + 2\pi n L(\alpha, \epsilon)$$

In Eq. (36), the symbol ψ_0 should be ϕ_0 so that the equation reads

$$\Delta u = D\{L(\psi) - L(\phi_0) - \cos\psi[M(\psi) - M(\phi_0)] - \sin\psi[N(\psi) - N(\phi_0)]\}$$

The authors also would like to take this opportunity to point out the physical significance of the right-hand side of the latter equation: the expression $L(\psi) - L(\phi_0)$ represents the decay of semimajor axis; the term $\cos\psi[M(\psi) - M(\phi_0)]$ represents the difference between apogee decay and perigee decay; and the term $\sin\psi[N(\psi) - N(\phi_0)]$ represents the short-periodic rotation of the line of apsides.

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